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Phase Report

November 1979



CHARACTERIZATION OF NONSTATIONARY CLUTTER

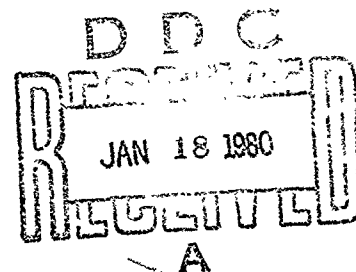
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PREFACE

The research on this program was conducted by the Radar Applications Division, Radar and Instrumentation Laboratory, Engineering Experiment Station, Georgia Institute of Technology, Atlanta, Georgia with Mr. J. L. Eaves serving as Project Director, and Mr. Donald J. Lewinski serving as Principal Investigator.

This report was prepared by the Engineering Experiment Station at the Georgia Institute of Technology for the U.S. Air Force, Rome Air Defense Center, under Contract F330602-78-C-0120. The Technical Monitor was Mr. William L. Simkins and Mr. J. T. Mastrangelo was the Contracting Officer. For the purposes of internal control at Georgia Tech, the effort was designated Project A-2199. The final report summarizes the work performed under this contract and gives recommendation for future work. This report covers work which was performed from 15 June 1978 to 30 September 1978.

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GLOSSARY

<u>Symbol</u>	<u>Definitions</u>
$p(z)$	Probability density of z
z	Instantaneous clutter power (random variable)
α_i	Random parameter
$p_1(z/\alpha_1, \alpha_2, \dots, \alpha_n)$	Conditional density of z given random parameters $(\alpha_1, \alpha_2, \dots, \alpha_n)$
$p_2(\alpha_1, \alpha_2, \dots, \alpha_n)$	Joint density of random parameters $(\alpha_1, \alpha_2, \dots, \alpha_n)$
a_i	Amplitude of i th scatterer contributing to z
ψ_i	Phase of i th scatterer contributing to z
r	Instantaneous clutter envelope (random variable)
k	Ratio of square of mean of z to variance of z
$\Gamma(k)$	Gamma function
$I_0(x)$	Modified Bessel function of first kind-order zero
u	Instantaneous mean of z (random variable)
u_2	Upper limit of u
u_1	Lower limit of u
a	$\frac{u_1}{u_2}$ = Inverse of dynamic range of u
$\gamma(b, x)$	Incomplete Gamma Function
$E_1(x)$	Exponential Integral
\bar{u}	Average Value of u
k_2	Ratio of square of mean of u to variance of u
$K_\nu(y)$	Modified Bessel function of second kind of order ν
$p_f(z^*)$	False alarm probability
z^*	Threshold value of z

GLOSSARY (continued)

<u>Symbol</u>	<u>Definitions</u>
y_2	Upper limit of $10 \log_{10} u$
y_1	Lower limit of $10 \log_{10} u$
σy^2	Variance of $10 \log_{10} u$
U_m	Median value of u

I. INTRODUCTION

Radar analysis and system design are usually performed under the assumptions that radar clutter is stationary and homogeneous; i.e., the probability density and moments (statistics) of the clutter power are assumed to be constant in time for a single resolution cell (stationary) and constant in space from resolution cell to resolution cell (homogeneous). In practice there are many situations which do not satisfy these conditions. For example, a single clutter cell composed of a single type of vegetation which is moving in the wind will have a first order density function and moments which vary with time and wind velocity. A radar viewing the ocean with sufficient resolution to resolve a single wave will record time varying density functions and mean values for a single resolution cell, depending on whether a wave is in the cell or not. A single cell composed of several different features (grass, trees, water, etc.) will have different statistics than a cell composed entirely of a single feature. Even when a resolution cell is composed of a single feature such as grass or trees, the variation in moisture content, number of scatterers per unit area, etc. result in an uncertainty in the density function and statistics. The feature or mixture of features in a resolution cell may also vary from cell to cell affecting histograms and moments of data gathered from a number of range and/or azimuth cells.

The examples delineated above indicate the need for a mathematical model to characterize the nonstationary and nonhomogeneity of radar clutter. Implied in the examples are practical radar problems such as cell averaging CFAR design and performance evaluation, accurate prediction of first order density functions of clutter power with attendant calculation of false alarm

and detection probabilities, prediction of spatial histograms, etc. In order to be useful, the model should have a firm theoretical basis, be relatively simple, and be relatable to experimental data.

II. APPROACH

The proposed technique for characterizing nonstationary and/or nonhomogeneous clutter is to treat the first order probability density as a function of random parameters. Thus if Z is the instantaneous value of clutter power and $\alpha_1, \alpha_2, \dots, \alpha_n$ are parameters, then the conditional density function of Z is $p_1(Z/\alpha_1, \alpha_2, \dots, \alpha_n)$. If the parameters are treated as random variables, then the density function for Z may be written as the N fold integral:

$$p(Z) = \iiint \dots p_1(Z/\alpha_1, \alpha_2, \dots, \alpha_n) p_2(\alpha_1, \alpha_2, \dots, \alpha_n) d\alpha_1 d\alpha_2 \dots d\alpha_n \quad (1)$$

where $p_2(\alpha_1, \alpha_2, \dots, \alpha_n)$ is the joint probability density of the random parameters.

Once the general formulation as expressed by Equation (1) has been assumed, the functional forms for P_1 and P_2 along with the type and number of parameters ($\alpha_1, \alpha_2, \dots, \alpha_n$) must be determined. Several possibilities for p_1 exist; however, the best choice appears to be the Nagakami density function, which was developed to model the fading statistics for high frequency propagation.

A. Choice of Conditional Density

It is shown in [1] that that Nagakami density is an approximate, general solution for the probability density function of the quantity:

$$r = \left| \sum_i a_i e^{j\psi_i} \right| \quad (2)$$

where a_i and ψ_i are random variables. Note that Equation (2) can be viewed

as the instantaneous envelope of clutter which is modeled as the sum of returns from individual discrete scatterers having amplitudes a_i and phases ν_i . Of course, the instantaneous clutter power s is just

$$s = r^2$$

The functional forms of the Nakagami density functions for $p(r)$ and $p(z)$ are:

$$p(r; k, u) = \frac{2(k)^k r^{2k-1} e^{-\frac{kr^2}{u}}}{\Gamma(k) (u)^k} \quad r \geq 0 \quad (3)$$

$$p(s; k, u) = \frac{(k)^k s^{k-1} e^{-\frac{ks}{u}}}{\Gamma(k) (u)^k} \quad s \geq 0 \quad (4)$$

where u is the mean or average value of $s = r^2$, k is the inverse of the normalized variance of s ,

$$k = \frac{[\text{mean}(s)]^2}{\text{Var}(s)} \quad (5)$$

and Γ denotes the gamma function

$$\Gamma(k) = \int_0^{\infty} t^{k-1} e^{-t} dt \quad k > 0$$

It is seen that Equation (4) has the general form of a gamma density function where the parameter k is a positive number greater than zero. For the Nakagami density functions, k is restricted to the interval $k > \frac{1}{2}$. In either

case Equations (3) and (4) can be viewed as families of curves with different functional forms depending on the parameter k . For $k = 1$, Equations (3) and (4) become Rayleigh and exponential densities, respectively. For $k = \frac{1}{2}$, Equation (3) becomes a single sided Gaussian density.

As derived in [1], Equation (4) corresponds to the Ricean density function

$$p(a) = \frac{\exp(-\frac{a^2}{2\sigma^2}) \exp(-\frac{A^2}{2\sigma^2}) I_0\left(\frac{\sqrt{a} A}{\sigma^2}\right)}{2\sigma^2} \quad (6)$$

for $\infty \geq k \geq 1$. In Equation (6) σ^2 is the variance of the random quadrature Gaussian components, A^2 is the steady power component and I_0 is the modified Bessel function of the first kind of order zero. For $1 \geq k \geq \frac{1}{2}$, Equation (4) corresponds to the Q density function.

$$p(a) = \frac{1}{\sqrt{\alpha\beta}} e^{-\frac{a}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)} I_0\left[\frac{a}{2}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)\right] \quad (7)$$

where

$$\alpha = 2\sigma^2(1 + \rho)$$

$$\beta = 2\sigma^2(1 - \rho)$$

σ^2 is the variance of the quadrature Gaussian components and ρ is the correlation coefficient between the quadrature components.

It is seen from the above discussion that the Gamma density function for the instantaneous clutter power as given by Equation (4) is justified from theoretical considerations and includes the exponential and Ricean density

functions which are frequently observed in experimental clutter data. The Gamma or Nakagami density function also simplifies the analysis because it is a function of only two parameters, the mean (u) and the inverse of the normalized variance (k). Treating the two parameters as random variables, the expression for the instantaneous power becomes

$$p(z) = \iint p_1(z/u, k) p_2(u, k) du dk \quad (8)$$

$$p(z) = \iint \frac{(k)^k u^{k-1} e^{-\frac{kz}{u}}}{\Gamma(k) (u)^k} p_2(u, k) du dk \quad (9)$$

where $p_2(u, k)$ is the joint probability density of u and k and the integrals are taken over the full ranges of allowable values of u and k .

B. Examples

The model for the first order density function as expressed by Equation (9) is directly applicable to the short and long time variation in the scattering from a given cell. For example, suppose that it is desired to model the terrain scattered return over an interval of wind velocities. It has been observed that for certain vegetation the functional forms of $p_1(z/\alpha_1, \alpha_2)$ and the mean of z vary with the wind velocity. Thus if the Nakagami density is a good representation of $p_1(z/\alpha_1, \alpha_2)$, then knowledge of k and u as a function of wind velocity would determine $p_1(z/u, k)$ in Equation (9); the frequency of occurrence of the various wind velocities would determine $p_2(u, k)$. Another single cell terrain example is the variation of the average power level of returns from vegetation depending on its moisture content, density, height, etc. In this case $p_1(z)$ is generally exponential ($k = 1$) and does

not change its functional form. Thus only the various values of the mean (u) and their frequency of occurrence for various conditions would be required to determine $p(z)$ from Equation (9).

Similar examples of clutter variation on a single cell basis occur for backscatter from the ocean surface. A low resolution radar will generally observe a first order density function that is exponential but has a time varying mean depending on sea state, wave height, wind velocity and direction, etc. This case would be treated in the same manner as the previously discussed case. High resolution radars recording returns from a single cell of the sea have observed "spiky" returns, which are attributed to the backscatter from individual waves, interleaved with returns which have an exponential first order density. In this case the overall density for $p(z)$ could be approximated by the discrete version of Equation (8):

$$p(z) = (1 - P) p_1(z/k_1, u_1) + P p_2(z/k_2, u_2) \quad (10)$$

where P is the probability that a wave is in the cell, $p_1(z/k_1, u_1)$ is the probability density of the clutter power when no wave is in the cell, and $p_2(z/k_2, u_2)$ is the probability density of the clutter power when a wave is in the cell.

The same techniques can be applied to histograms obtained from samples taken from different range and/or azimuth cells. Assume that samples from N independent cells are obtained and that the density associated with the n th cell is $p(z/u_n, k_n)$. The overall histogram can then be written as

$$p(z) = \sum_n \frac{1}{N} p(z/u_n, k_n) \quad (11)$$

When all cells having the same values of u and k are combined, Equation (11) becomes

$$p(z) = \sum_i \sum_j P(u_i, k_j) p(z/u_i, k_j) \quad (12)$$

where $P(u_i, k_j)$ is the fractional number of cells with parameters u_i and k_j . In the limit as the values of u and k become continuous, the sum indicated in Equation (12) approaches the double integral in Equation (8).

C. Investigations

The application of the suggested theory and techniques should follow two approaches: analytical and experimental. In the analytical area $p(z)$ should be obtained through Equation (9) for representative analytically tractable forms of $p_2(u, k)$. Experimental data should be scrutinized to determine the validity of the Nakagami density to approximate the conditional density $p_1(z/\alpha_1, \alpha_2, \alpha_n)$ and to obtain $p_2(\alpha_1, \alpha_2, \dots, \alpha_n)$ for various cases of interest.

III. INITIAL RESULTS

A. Analysis

In order to obtain some meaningful results during the initial phase of the program, it was decided to investigate the effects on the first order density, $p(z)$, caused only by variations in the mean. Hence the parameter k was assumed to be constant in Equation (9). The first order density can then be written as

$$p(z) = \int_0^{\infty} \frac{(k)^k z^{k-1} e^{-\frac{kz}{u}}}{\Gamma(k) (u)^k} p_2(u) du \quad (13)$$

where $p_2(u)$ is the probability density of the mean or average value. From Equation (13) $p(z)$ was determined for various representative functions, $p_2(u)$. Selected plots were made to show the general effects of $p_2(u)$ on $p(z)$. The $p_2(u)$ densities which were investigated included the Gamma, uniform, log uniform, and log normal.

1. Uniform

A frequently used analytical density function is the uniform density

$$\begin{aligned} p_2(u) &= \frac{1}{u_2 - u_1} & u_2 \geq u \geq u_1 \\ p_2(u) &= 0 & u < u_1, u > u_2 \end{aligned} \quad (14)$$

When Equation (14) is substituted into Equation (13),

$$p(a) = \frac{1}{u_2 - u_1} \int_{u_1}^{u_2} \frac{(k)^k a^{k-1} e^{-\frac{ka}{u}}}{\Gamma(k) (u)^k} du \quad (15)$$

Through the change in variable $t = \frac{ka}{u}$, Equation (15) becomes

$$p(a) = \frac{k}{(u_2 - u_1) \Gamma(k)} \int_{\frac{ka}{u_2}}^{\frac{ka}{u_1}} t^{k-2} e^{-t} dt \quad (16)$$

The Incomplete Gamma function, which is tabulated in several sources, is defined as

$$\gamma(a, x) = \int_0^x t^{a-1} e^{-t} dt \quad (17)$$

for $a > 0$. Thus Equation (16) can be written as

$$p(a) = \frac{k}{(u_2 - u_1) \Gamma(k)} \left[\gamma(k-1, \frac{ka}{u_1}) - \gamma(k-1, \frac{ka}{u_2}) \right] \quad (18)$$

for $k > 1$. For the special case of $k = 1$, Equation (16) becomes

$$p(a) = \frac{1}{u_2 - u_1} \int_{\frac{a}{u_2}}^{\frac{a}{u_1}} \frac{e^{-t}}{t} dt \quad (19)$$

In terms of the exponential integral:

$$E_1(x) = \int_x^{\infty} \frac{e^{-t}}{t} dt \quad (20)$$

Equation (19) can be expressed as

$$p(s) = \frac{1}{u_2 - u_1} \left[E_1\left(\frac{s}{u_2}\right) - E_1\left(\frac{s}{u_1}\right) \right] \quad (21)$$

for $k = 1$.

In order to determine the effects on $p(s)$ of the spread in the mean u , values of $p(s)$ were calculated from Equation (21) for dynamic ranges $\frac{u_2}{u_1} = \frac{1}{a}$ of 1.11, 2, and 10. These values are listed in Table 1 and plotted in Figure 1. For comparison purposes, it was decided to plot the quantity $u_2 p(s)$ versus the normalized variable $\frac{s}{u_2}$. Another, possibly more useful, plot would show $\bar{u} p(s)$ versus $\frac{s}{\bar{u}}$ where \bar{u} is the mean of u and is related to u_1 and u_2 through the relationships:

$$u_2 = \frac{2\bar{u}}{1+a}$$

$$u_1 = \frac{2a\bar{u}}{1+a}$$

Figure 1 and the data in Table I indicate that for small spreads in the mean (a large, $\frac{u_2}{u_1} = \frac{1}{a}$ small), $p(s)$ approaches the function $\frac{e^{-\frac{s}{u_2}}}{u_2}$. As the spread in the mean u increases, the function of $u_2 p(s)$ becomes more concentrated near the origin. These results are to be expected, because as a approaches 1.0, u_1 approaches u_2 and p_2 approaches a delta function centered

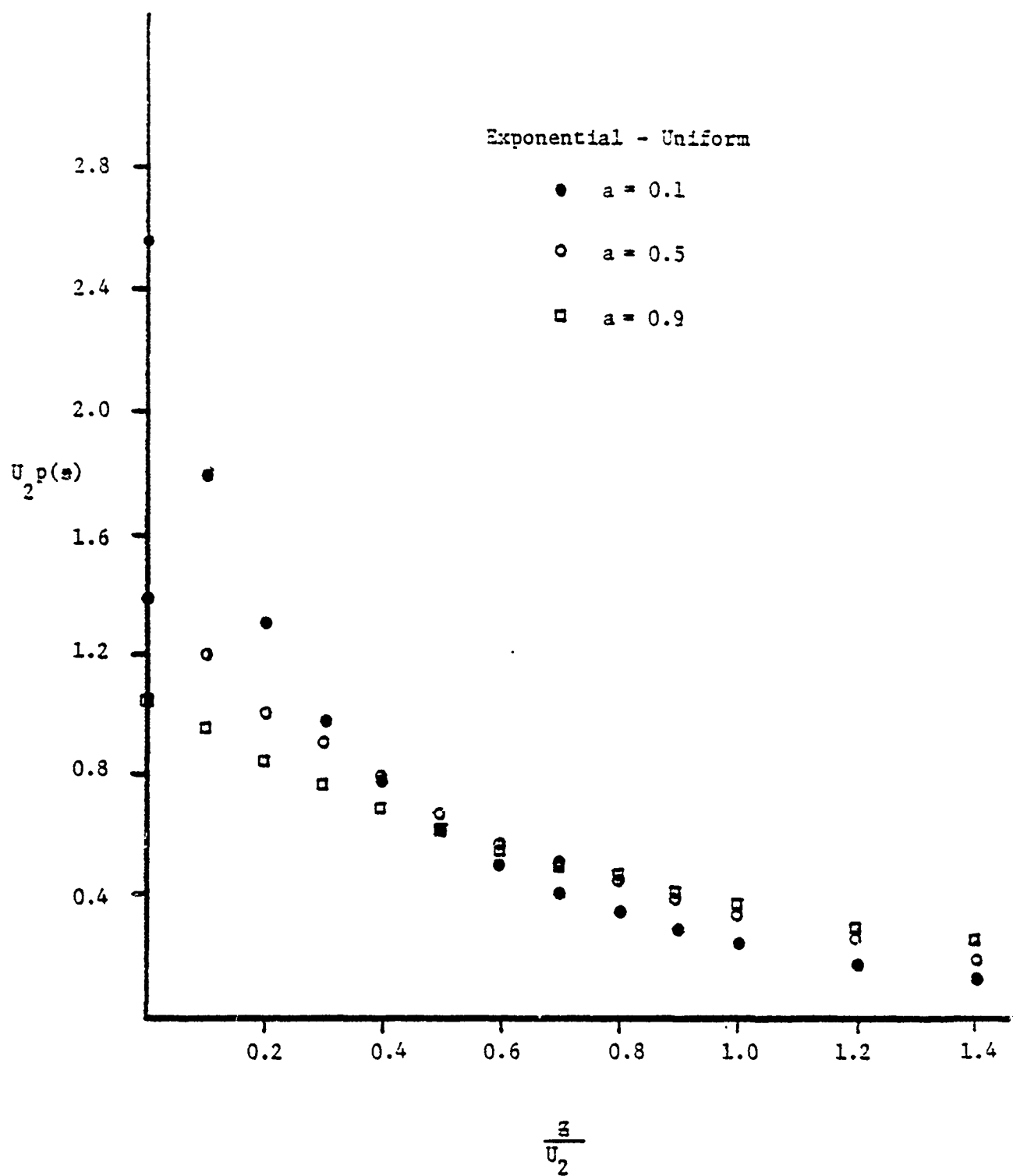


Figure III-1. Normalized Probability Density Function of Clutter Power for Exponential - Uniform.

$\frac{z}{U_2}$	$a = 0.9$ $U_2 p(z)$	$a = 0.5$ $U_2 p(z)$	$a = 0.1$ $U_2 p(z)$	$a \rightarrow 0$ $U_2 p(z)$
0	1.054	1.386	2.558	
.1	.948	1.201	1.782	1.823
.2	.853	1.041	1.304	1.223
.3	.768	.903	0.992	.905
.4	.690	.784	0.776	.702
.5	.622	.681	0.621	.560
.6	.560	.592	0.504	.454
.7	.504	.515	0.415	.374
.8	.450	.449	0.345	.311
.9	.408	.391	0.289	.260
1.0	.367	.341	0.244	.219
1.1	.330	.297	.206	.186
1.2	.297	.260	.176	.158
1.3	.267	.227	.1505	.135
1.4	.241	.199	.129	.116
1.5	.217	.174	0.111	.100
2.0	.128	.090	0.054	.0489

Table III-1. $U_2 p(z)$ As Function of $\frac{z}{U_2}$ For Exponential

$p_1(z/u)$ and Uniform $p_2(u)$

at $u = u_2$. The resulting expression for $p(z)$ is then

$$p(z) = \int_0^{\infty} \frac{e^{-\frac{z}{u}}}{u} \delta(u - u_2) du = \frac{e^{-\frac{z}{u_2}}}{u_2}$$

As z approaches zero, u_1 approaches zero and Equation 19 becomes

$$p(z) = \frac{1}{u_2} \int_{\frac{z}{u_2}}^{\infty} \frac{e^{-t}}{t} dt = \frac{1}{u_2} E_1\left(\frac{z}{u_2}\right) \quad (22)$$

The expression in Equation (22) approaches infinity at $\frac{z}{u_2} = 0$ and has the largest negative slope for all values of z between zero and one.

2. Gamma

Since the Gamma density encompasses a relatively wide family of functions, it is instructive to investigate the behavior of $p(z)$ when $p_2(u)$ is Gamma distributed. In this case

$$p_2(u) = \frac{(k_2)^{k_2} u^{k_2-1} e^{-\frac{k_2 u}{\bar{u}}}}{\Gamma(k_2) (\bar{u})^{k_2}} \quad (23)$$

where \bar{u} is the mean of u and k_2 is the inverse of the normalized variance of u . Using Equation (13) the expression for $p(z)$ becomes

$$p(z) = \frac{(k)^k (k_2)^{k_2} z^{k-1}}{\Gamma(k) \Gamma(k_2) (\bar{u})^{k_2}} \int_0^{\infty} \frac{e^{-\left(\frac{k_2 u}{\bar{u}} + \frac{kz}{u}\right)}}{u^{k-k_2+1}} du \quad (24)$$

By the change of variable $\chi = \frac{k_2 u}{\bar{u}}$, the integral in Equation (24) can be written as

$$\int_0^\infty \frac{e^{-\left(\frac{k_2 u}{\bar{u}} + \frac{k a}{u}\right)}}{u^{k-k_2+1}} du = \frac{(k_2)^{k-k_2}}{(\bar{u})^{k-k_2}} \int_0^\infty \frac{e^{-\left(\chi - \frac{k a k_2}{\chi \bar{u}}\right)}}{\chi^{k-k_2+1}} d\chi \quad (25)$$

Using the identity

$$\int_0^\infty \frac{e^{-(\chi + \frac{y^2}{4\chi})}}{\chi^{\nu+1}} d\chi = \frac{2^{\nu+1}}{y^\nu} K_\nu(y)$$

in Equation (25) and substituting the resultant into Equation (24), the expression for $p(a)$ becomes

$$p(a) = \frac{\frac{k+k_2}{2} \frac{k+k_2-2}{2} (a)}{\Gamma(k) \Gamma(k_2) (\bar{u})^{\frac{k_2+k}{2}}} K_{k_2-k} \left(\sqrt{\frac{4k_2 a}{\bar{u}}} \right) \quad (26)$$

For the special case of $k = 1$, $p_1(z/u)$ is exponential,

$$p(a) = \frac{\frac{k_2+1}{2} \frac{k_2-1}{2} (a)}{\Gamma(k_2) (\bar{u})^{\frac{k_2+1}{2}}} K_{k_2-1} \left(\sqrt{\frac{4k_2 a}{\bar{u}}} \right) \quad (27)$$

The density function given by Equation (27) is especially interesting because it can be integrated in closed form to find the false alarm probability:

$$p_{\bar{z}}(z^*) = \int_{z^*}^{\infty} p(z) dz = 1 - \int_0^{z^*} p(z) dz \quad (28)$$

Substituting Equation (27) into Equation (28)

$$p_{\bar{z}} = 1 - \frac{2(k_2)^{\frac{k_2+1}{2}}}{\Gamma(k_2) (\bar{u})^{\frac{k_2+1}{2}}} \int_0^{z^*} (z)^{\frac{k_2-1}{2}} K_{k_2-1} \left(\sqrt{\frac{4k_2 z}{\bar{u}}} \right) dz \quad (29)$$

Let $t = \sqrt{\frac{4k_2 z}{\bar{u}}}$, then

$$p_{\bar{z}} = 1 - \frac{1}{\Gamma(k_2) (2)^{k_2-1}} \int_0^{\frac{\sqrt{4k_2 z^*}}{\sqrt{\bar{u}}}} t^{k_2} K_{k_2-1}(t) dt \quad (30)$$

Making use of the identity

$$t^{\nu} K_{\nu-1}(t) dt = 2^{\nu-1} \Gamma(\nu) - \chi^{\nu} K_{\nu}(\chi) \quad (31)$$

Equation (30) becomes

$$P_{\bar{F}}(s^*) = \frac{2(k_2 s^*)^{\frac{k_2}{2}} K_{k_2} \left(\sqrt{\frac{4k_2 s^*}{\bar{u}}} \right)}{\Gamma(k_2) (\bar{u})^{\frac{k_2}{2}}} \quad (32)$$

Thus for the special case of an exponential density for $p_1(s/u)$ and a Gamma density with parameters u and k_2 for $p_2(u)$, the false alarm probability for a given power threshold of s^* is given by Equation (32).

Figure 2 is a plot of Equation (27) for values of k_2 equal to 1.5 and 3. Table 2 lists the values used in the plot. The behavior is similar to that shown in Figure 1. As k_2 becomes large ($k_2 = 3$), the variance of u about its mean \bar{u} becomes small and $p(z)$ approaches the exponential density. As k_2 becomes smaller, larger spread of the variable u about its mean, \bar{u} $p(z)$ attains a higher maximum at $\frac{s}{\bar{u}} = 0$ and falls off more rapidly as a function of $\frac{s}{\bar{u}}$ for small values of the argument. These observations are verified by the limiting forms of k_2 . As k_2 approaches infinity, $p_2(u)$ approaches $\delta(u - \bar{u})$ and $p(z)$ approaches $\frac{e^{-\frac{s}{\bar{u}}}}{\bar{u}}$. As k_2 approaches the limiting value of $\frac{1}{2}$ in the Nakagami density function,

$$p(s) \rightarrow \frac{e^{-\sqrt{\frac{2s}{\bar{u}}}}}{\sqrt{2\bar{u}} \sqrt{s}} \quad (33)$$

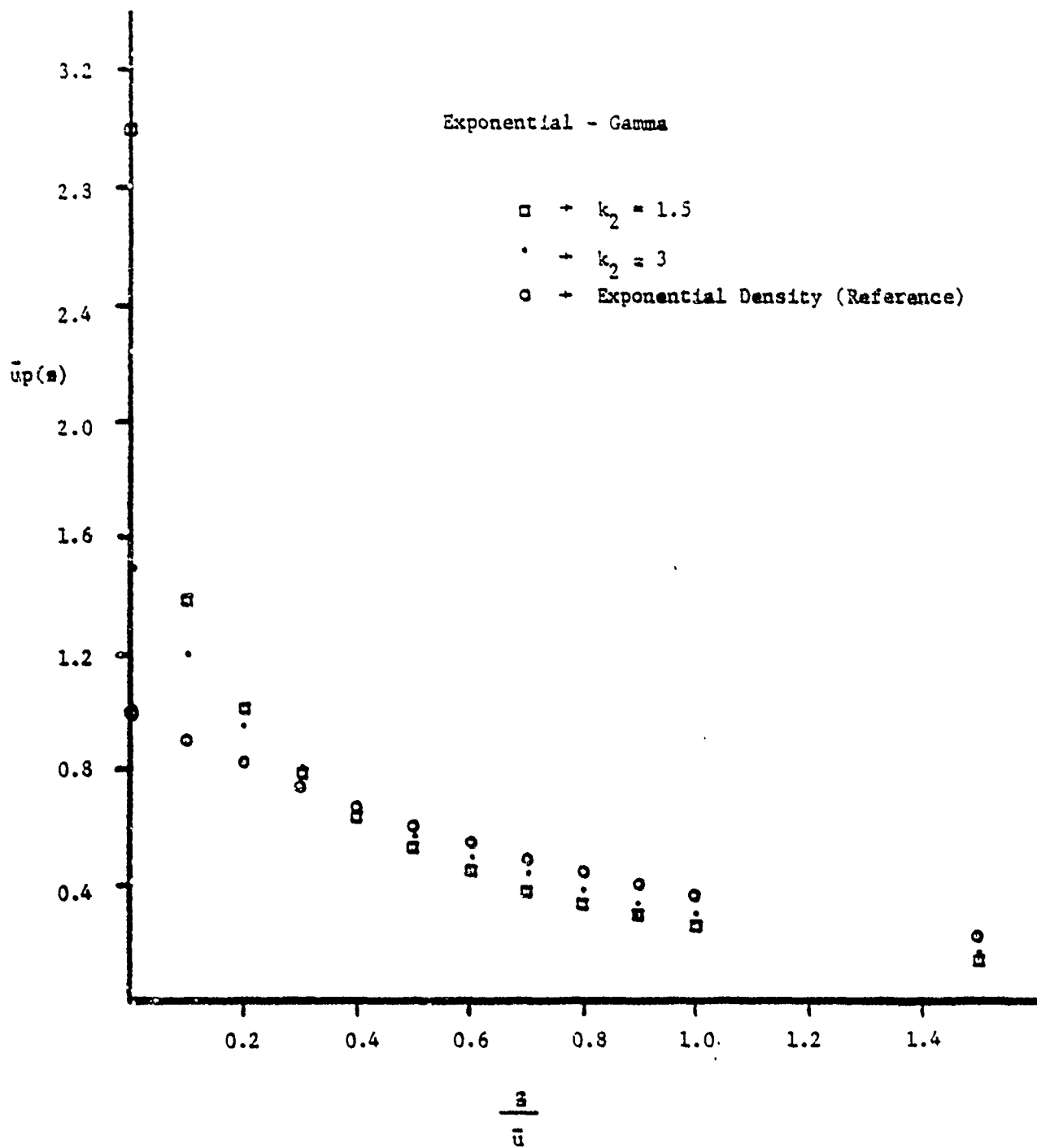


Figure III-2. Normalized Probability Density Function of Clutter Power for Exponential - Gamma.

$\frac{z}{u}$	$k_2 \approx 1.5$ $\bar{u}p(z)$	$k_2 \approx 3.0$ $\bar{u}p(z)$
0	3	1.50
0.01	2.35	1.46
0.1	1.38	1.18
0.2	1.003	.962
0.3	.784	.805
0.4	.637	.684
0.5	.531	.587
0.6	.450	.509
0.7	.386	.444
0.8	.335	.389
0.9	.294	.343
1.0	.259	.304
1.5	.149	.175
2.0	.094	.107

Table III-2. $\bar{u}p(z)$ As Function of $\frac{z}{u}$ For Exponential $p_1(z/u)$ and Gamma $p_2(u)$.

It is interesting to note that an independent analysis of histograms obtained from range swept radar data [2] indicated that $p(z)$ had the form

$$p(z) = \frac{c e^{-b \sqrt{z}}}{\sqrt{z}} \quad (34)$$

which is the same form as Equation (33).

3. Log Uniform

Radar backscatter data is usually obtained in the form of $K \log z$ instead of direct measurement of the quantity z . Thus it is useful to investigate density functions for $p_2(u)$ which are distributed in terms of $\log u$. Consider the density function

$$p(y) = \frac{1}{y_2 - y_1} \quad y_2 \geq y \geq y_1 \quad (35)$$

where $y = 10 \log_{10} u$. The density for u then has the form

$$p_2(u) = \frac{1}{u \ln \frac{u_2}{u_1}} \quad u_2 > u > u_1 \quad (36)$$

where $u_2 = (10)^{\frac{y_2}{10}}$ and $u_1 = (10)^{\frac{y_1}{10}}$. The density function given by Equation (36) is the log uniform density function.

When the log uniform density function for $p_2(u)$ is substituted into Equation (13)

$$p(a) = \int_{u_1}^{u_2} \frac{(k)^k z^{k-1} e^{-\frac{ka}{u}}}{\Gamma(k) (u)^{k+1} \ln \left(\frac{u_2}{u_1} \right)} du \quad (37)$$

For the case $k = 1$

$$p(a) = \frac{1}{\ln \left(\frac{u_2}{u_1} \right)} \int_{u_1}^{u_2} \frac{e^{-\frac{a}{u}}}{u^2} du \quad (38)$$

Equation (38) can be readily integrated and it is found that

$$p(a) = \frac{e^{-\frac{a}{u_2}} - e^{-\frac{a}{u_1}}}{a \ln \left(\frac{u_2}{u_1} \right)} \quad (39)$$

Equation (39) is plotted in Figure 3 from the computed values shown in Table 3. The parameter a is defined in the same manner as that used for uniform density: $u_1 = au_2$. It is seen that the behavior of $p(z)$ for log uniform $p_2(u)$ is similar to that for uniform and Gamma $p_2(u)$. For small spreads of $p_2(u)$, $a \approx 1.0$, $p(a)$ approaches the exponential density. Large spreads in $p_2(u)$, $a \approx 0$, causes $u_2 p(a)$ to attain large peaks at $\frac{a}{u_2} = 0$ and to fall off rapidly as $\frac{a}{u_2}$ increases for small values of $\frac{a}{u_2}$.

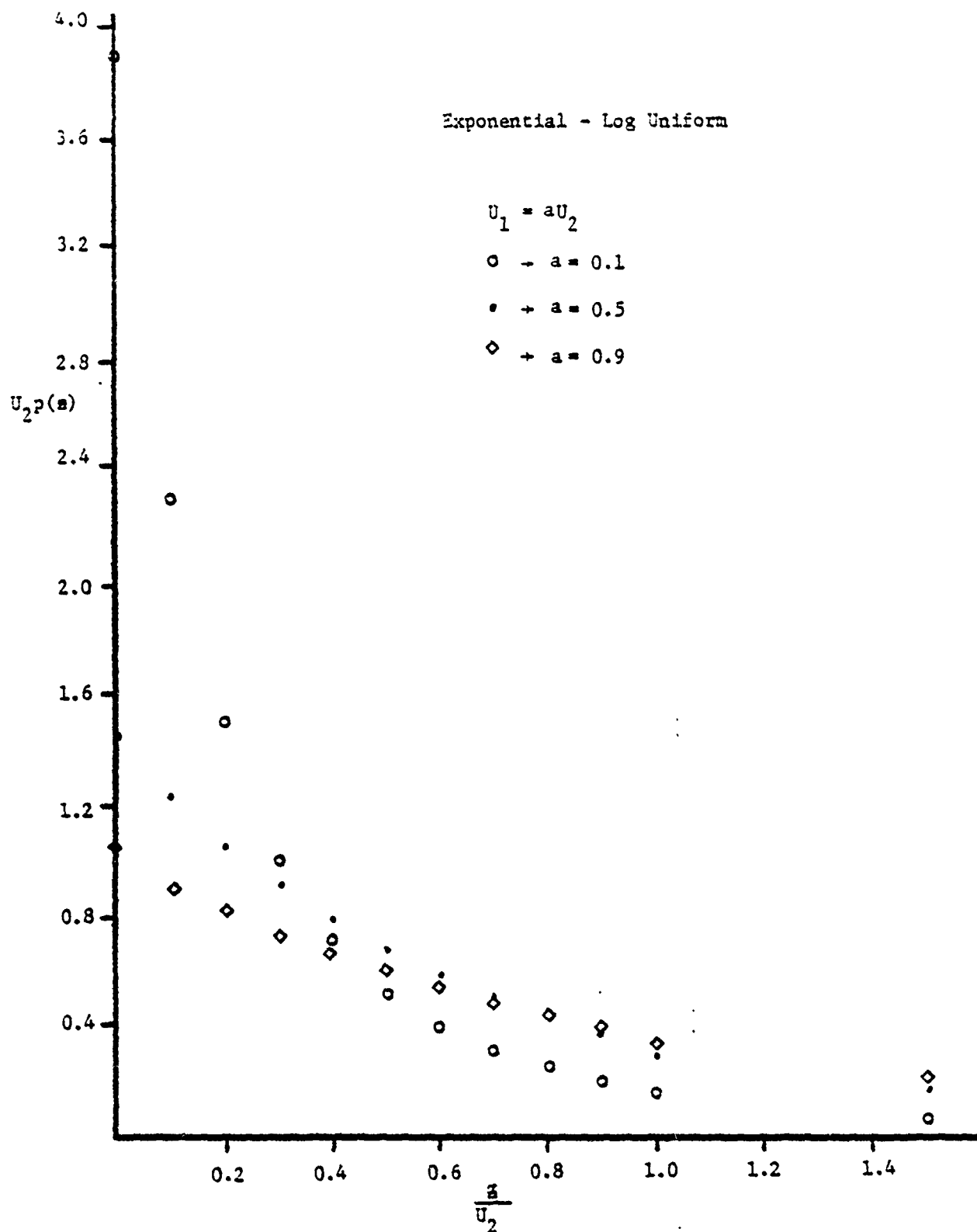


Figure III-3. Normalized Probability Density Function of Clutter Power for Exponential - Log Uniform.

$\frac{z}{U_2}$	$a = 0.9$ $U_2 p(z)$	$a = 0.5$ $U_2 p(z)$	$a = 0.1$ $U_2 p(z)$
0	1.055	1.44	3.91
.1	.949	1.24	2.33
.2	.854	1.07	1.48
.3	.768	.923	1.00
.4	.691	.797	.708
.5	.622	.689	.521
.6	.560	.595	.395
.7	.504	.515	.308
.8	.453	.446	.244
.9	.408	.387	.196
1.0	.367	.335	.160
1.5	.217	.167	.0646
3.0	.045	.0228	.0072

Table III-3. $U_2 p(z)$ As Function of $\frac{z}{U_2}$ For Exponential $p_1(z/u)$
and Log Uniform $p_2(u)$.

4. Log Normal

Another commonly used density is the log normal density, whereas the name implies, the log of the variable is normally distributed. Thus for $y = 10 \log_{10} u$,

$$p(y) = \frac{e^{-\frac{(y-\bar{y})^2}{2\sigma_y^2}}}{\sqrt{2\pi} \sigma_y} \quad (40)$$

where \bar{y} and σ_y^2 are the mean and variance of y , respectively. The density function for u is then

$$p_2(u) = \frac{e^{-\frac{(\ln u - \overline{\ln u})^2}{2\sigma^2}}}{u \sqrt{2\pi} \sigma} \quad \infty \geq u \geq 0 \quad (41)$$

where $\overline{\ln u}$ and σ^2 are the mean and variance of $\ln u$, respectively. From Equation (13) the density function for the clutter power becomes

$$p(s) = \int_0^\infty \frac{(k)^k s^{k-1} e^{-\frac{ks}{u}} e^{-\frac{(\ln u - \overline{\ln u})^2}{2\sigma^2}}}{\sqrt{2\pi} \sigma \Gamma(k) u^{k+1}} du \quad (42)$$

which reduces to

$$p(z) = \int_0^{\infty} \frac{e^{-\frac{z}{u}} e^{-\frac{(\ln u - \overline{\ln u})^2}{2\sigma^2}}}{\sqrt{2\pi} \sigma u^2} du \quad (43)$$

for $k = 1$. Thus far attempts to obtain closed form expressions for the integrals in Equations (42) and (43) have not been successful, although series representations for the two equations have been derived.

Nakagami [1] states without proof that the density function for $\tau = 10 \log_{10} z$, when u is log normal and $p_1(z/u)$ is a Gamma density is given by the expression

$$p_{\tau}(\tau) = F(\tau, k, \bar{\tau}_0) S(\tau, k, \bar{\tau}_0) \quad (44)$$

where

$$F(\tau, k, \bar{\tau}_0) = \frac{2 (k)^k}{m \Gamma(k)} \exp \left[\frac{2k(\tau - \bar{\tau}_0)}{m} - k e^{\frac{2(\tau - \bar{\tau}_0)}{m}} \right]$$

$$S(\tau, k, \bar{\tau}_0) = \frac{m}{m^2 + 4 \sigma_0^2 kQ} \exp \left[\frac{2 \sigma_0^2 k^2 (1-Q)^2}{m^2 + 4 \sigma_0^2 kQ} \right]$$

and

$$\begin{aligned} \tau_0 &= 10 \log_{10} u \\ \bar{\tau}_0 &= E(10 \log_{10} u) = \text{mean value of } 10 \log_{10} u \end{aligned}$$

$$\sigma_o^2 = \text{Variance of } 10 \log_{10} u$$

$$Q = e^{\frac{1}{m}(\tau - \tau_o)}$$

$$m = 20 \log e = 8.686$$

Transforming to the variable $z = (10)^{\frac{\tau}{10}} = e^{\frac{\tau \ln 10}{10}}$, it is found that

$$p(z) = \frac{10}{z \ln 10} p_{\tau} \left(\frac{10 \ln z}{\ln 10} \right)$$

$$= \frac{\left(\frac{ks}{u_m} \right)^k e^{-\frac{ks}{u_m}} \exp \left[\frac{k^2 \sigma^2 \left(1 - \frac{s}{u_m} \right)^2}{2 \left(1 + k \sigma^2 \frac{s}{u_m} \right)} \right]}{z \Gamma(k) \sqrt{1 + k \sigma^2 \frac{s}{u_m}}} \quad (45)$$

where

$$\sigma^2 = \text{variance of } \ln u$$

$$u_m = \text{median value of } u = e^{\overline{\ln u}}$$

$$\overline{\ln u} = \text{mean value of } \ln u$$

For the special case of $k = 1$,

$$p(s) = \frac{e^{-\frac{s}{u_m}} \exp \left[\frac{\sigma^2 \left(1 - \frac{s}{u_m} \right)^2}{2 \left(1 + \sigma^2 \frac{s}{u_m} \right)} \right]}{u_m \sqrt{1 + \sigma^2 \frac{s}{u_m}}} \quad (46)$$

Equation (46) is plotted in Figure 4 as a function of $\frac{s}{u}$ for $\sigma^2 = 0.5$ and 1.5.

The quantity plotted is $\bar{u} p(s)$. The values used in the plots are given in

Table 4. As the spread in u becomes small (σ^2 is small) Equation (45) approaches the function

$$p(z) = \frac{\left(\frac{kz}{u_m}\right)^k e^{-\frac{kz}{u_m}}}{z \Gamma(k)} \quad (47)$$

Since

$$\bar{u} = e^{\frac{\sigma^2}{2}} e^{\overline{\ln u}} = u_m e^{\frac{\sigma^2}{2}},$$

\bar{u} approaches u_m as σ^2 becomes small and

$$\lim_{\sigma \rightarrow 0} p(z) = \frac{\left(\frac{kz}{\bar{u}}\right)^k e^{-\frac{kz}{\bar{u}}}}{z \Gamma(k)} \quad (48)$$

For $k = 1$,

$$\lim_{\sigma \rightarrow 0} p(z) = \frac{e^{-\frac{z}{\bar{u}}}}{\bar{u}} \quad (49)$$

Thus as $\sigma \rightarrow 0$, $p(z)$ is approximately equal to $p_1(z/u)$ evaluated at $u = \bar{u}$ as can be seen from Equation (13). This result agrees with the plot for small

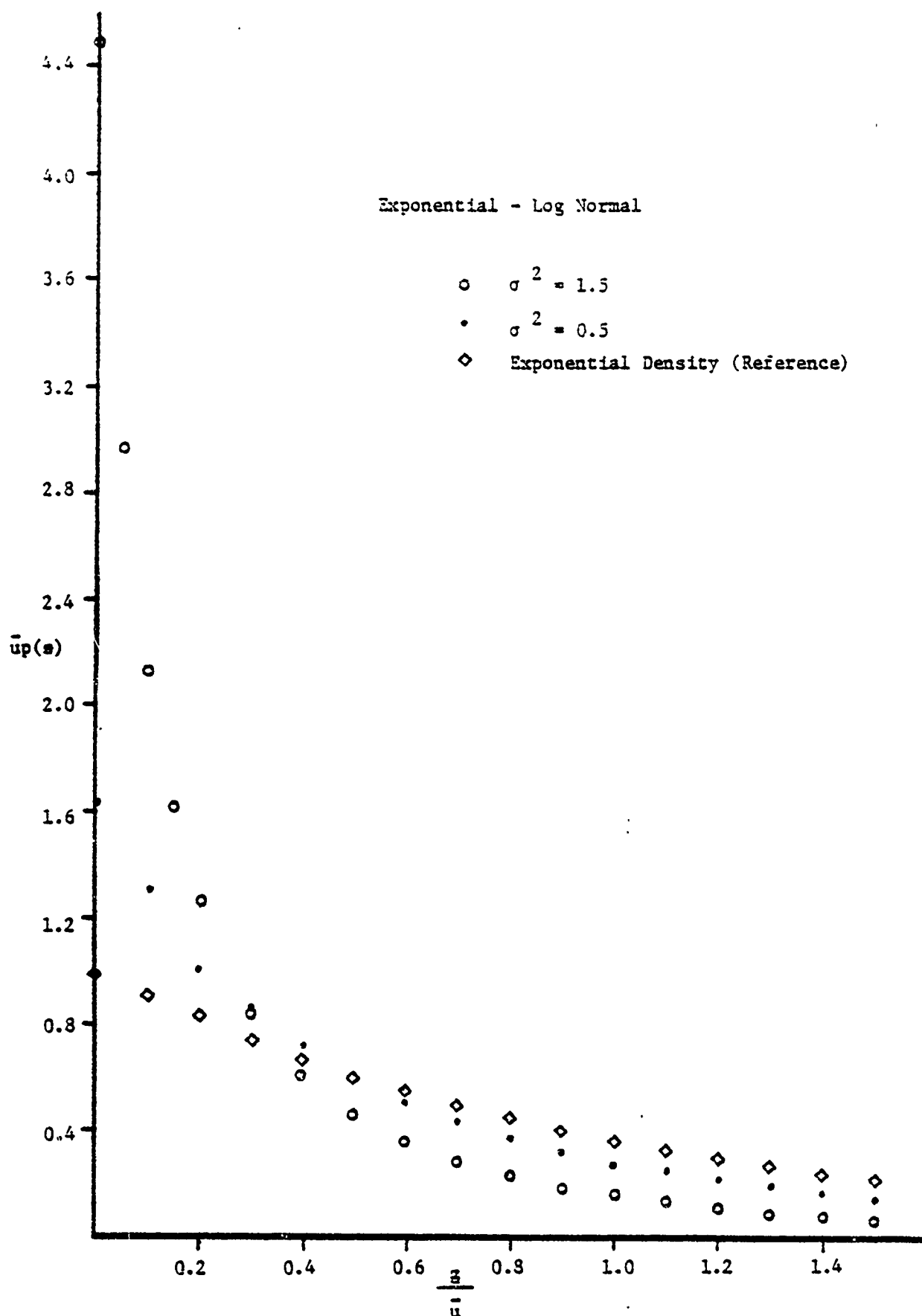


Figure III-4. Normalized Probability Density Function of Clutter Power for Exponential - Log Normal.

$\frac{a}{u}$	$\sigma^2 = 0.5$ $\bar{u}_p(a)$	$\sigma^2 = 1.5$ $\bar{u}_p(a)$
0	1.65	4.48
.01	1.61	4.10
.1	1.31	2.13
.2	1.06	1.26
.3	.866	.845
.4	.718	.607
.5	.602	.457
.6	.510	.355
.7	.435	.283
.8	.374	.229
.9	.323	.188
1.0	.281	.156
1.1	.246	.131
1.2	.215	.110
1.3	.190	.0934
1.4	.168	.0797
1.5	.149	.0682
2.0	.0853	.0329
3.0	.0319	.00876
4.0	.0133	.00254
5.0	.00583	.000773
10.0	.000141	2.63×10^{-6}

Table III-4. $\bar{u}_p(a)$ As Function of $\frac{a}{u}$ For Exponential $p_1(a/u)$
and Log Normal $p_2(u)$.

$\sigma^2 = 0.5$). For larger spreads in u (σ^2 large), Equation (46) indicates that at the point $z = 0$,

$$p(0) = \frac{e^{\frac{\sigma^2}{2}}}{u} = \frac{e^{\frac{\sigma^2}{2}}}{u} \quad (50)$$

Thus $\bar{u} p(z)$ at $z = 0$ becomes large as σ^2 becomes large. This trend is verified by the plot for $\sigma^2 = 1.5$. It is seen that the general behavior of $p(z)$ for log normal u is similar to that for the other $p_2(u)$ densities discussed earlier.

Nakagami [1] indicates that when $p_2(u)$ is log normal and σ^2 is large, $p(z)$ approaches a log normal density when $p_1(z/u)$ is a Gamma density. For $k = \frac{1}{2}$ he states that $p(z)$ is log normal when $\sigma_0 > 10$ which corresponds to $\sigma > 2.3$; for $k = 1$, $p(z)$ is said to be log normal for $\sigma_0 > 7.0$ or $\sigma > 1.61$. The log normal density has been proposed for terrain at low grazing angles [3].

B. Data Evaluation

A limited amount of experimental data was visually inspected to obtain the appropriate form of $p_2(u)$. It appears that the variation in the average value of the backscatter from trees at frequencies of 9.5 GHz, 16.5 GHz, 35 GHz, and 95 GHz may be approximated by either log uniform or truncated log normal densities. Extensive reduction of experimental data for the feature types of interest at the desired frequencies, depression angles, time of year, etc. would be required to properly determine the correct functional form of $p_2(u)$.

IV. SUMMARY

The need for a statistical model to describe nonstationary and nonhomogeneous radar clutter returns is discussed in Section I. Examples are given which indicate that when the variability and uncertainty of clutter are taken into account, nonstationarity and nonhomogeneity of the returns are the rules instead of the exceptions.

In Section II the general and specific approaches to the problem are outlined. The general approach is to model the first order density function of the instantaneous power of the clutter return as the expected value of a conditional density which is a function of random parameters. The expected value is taken over the full ranges of all the random parameters. If n parameters are involved, then the conditional density as a function of each of the parameters must be known along with the joint density of the parameters. An n fold integral is then required to obtain the first order density function of the instantaneous clutter power.

It is shown that the Nakagami or Gamma density is a good choice for the conditional density function for several reasons. It has valid theoretical and practical justifications and is a function of only two parameters: the mean (u) and the inverse of the normalized variance (k). The Nakagami density includes the exponential and Ricean densities as special cases; its particular functional form depends on the value of k .

The analysis in Section III considers the case where the conditional density has the Nakagami form and only the mean u is a random variable; the parameter k is assumed to be a constant. Although somewhat restrictive, variation only in the mean does occur in practice and it also simplifies

the analysis. Four separate density functions for the mean are considered: uniform, Gamma, log uniform, and log normal. General expressions are derived and selected plots are made for the first order density of the clutter power (s) for each distribution of the mean.

When the mean is uniformly distributed, $p(s)$ can be expressed, see Equation (18), as the difference between two incomplete Gamma functions for $k > 1$. For $k = 1$, exponential density for $p(s/u)$, $p(s)$ is given by Equation (21) as the difference between two exponential integrals.

For a mean having a Gamma density, the general expression for $p(s)$ as indicated by Equation (26) is a function of \bar{u} (the average value of u) and k_2 (the inverse of the normalized variance of u); it also contains as a factor a modified Bessel function of the second kind of order $k_2 - k$. When $k = 1$, $p(s)$ can be integrated in closed form to obtain the false alarm probability, P_f , which is a function of k_2 , \bar{u} and the threshold level s^* .

A log uniform density for u when $k = 1$ results in an expression for $p(s)$, Equation (39), which is proportional to the difference between two exponentials and inversely proportional to s . The exponentials are functions of the upper and lower limits of u .

When the mean has a log normal density the integral expression for $p(s)$ is difficult to evaluate in closed form. Some results which are stated without proof in [1] were used to obtain an expression for $p(s)$, Equation (45), which is a complicated function of k , u_m (the median value of u) and σ^2 (the variance of $\ln u$).

For the case where the conditional density $p_1(s/u)$ is exponential, the general behavior of $p(s)$ for all of the assumed functional forms of $p_2(u)$ give similar results. When the spread in the mean is small, the density for

$p(s)$ is approximately equal to $p_1(s/u)$ evaluated at $u = \bar{u}$. As the spread in u becomes large, the function $\bar{u} p(s)$ becomes highly concentrated at $s = 0$.

Preliminary visual inspection of a limited amount of experimental data on the backscatter from trees seems to indicate that the density for the mean clutter power, $p_2(u)$, can be approximated by log uniform or truncated log normal densities. In order to obtain reliable estimates of $p_2(u)$, histograms of experimental data on the average backscatter for the terrains of interest at the desired frequencies, depression angles, polarizations, etc. are required.

V. FUTURE WORK

Future efforts on the characterization of nonstationary clutter should include theoretical and experimental investigations. Several analytical problems are suggested based on the effort to date. First of all the results given in this report for a mean which is log normally distributed are based on an equation in a technical paper that is stated without proof. Since clutter with log normal mean has been measured experimentally, an additional effort should be made to obtain a rigorous derivation. Another appropriate theoretical topic would be to perform an analysis for the parameter k in the Nakagami density function similar to that reported here for the parameter u . The approach would be to assume various standard densities for $p_2(k)$ and integrate the expression $p_1(s/k)p_2(k)$ to obtain the probability density function for the instantaneous clutter power $p(s)$. Other theoretical topics would be suggested from the analysis of experimental data. For example, if the data indicate that the assumed models for $p_1(s/u)$ and $p_2(u)$ are inadequate, then additional derivations similar to those reported here would be required.

In the area of experimental data analysis the task would include the generation of histograms to determine for various terrains the conditional density and the densities for the mean and normalized variance. The best fit of a Nakagami density to the measured conditional density would be made to determine the adequacy of the present model of $p_1(s/u, k)$. If the Nakagami density is found to be valid, then temporal or spatial histograms of the data would be compared to $p(s)$ as predicted through the use of $p_1(s/u, k)$ and $p_2(u, k)$ to determine the validity of the overall model.

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